PROBLEMY KOLEJNICTWA
RAILWAY REPORTS Zeszyt 180 (September 2018) ISSN 0552-2145 (print) ISSN 2544-9451 (on-line)

Optimization of the Efficiency of Braking Energy Recovery in Rail Transport by Changing Arrival Time

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Summary

The article refers to the previous work of the authors, in which the model of traffic organization of cooperating trains including the optimization of the use of energy returned to the catenary was presented. In the presented article, the model was modified by changing the main control variable, which affects the efficient use of energy. Departure time was changed for the arrival time of the train to the stop or station. The optimization is done by controlling the arrival time to the station in the acceptable (scheduled) range while maintaining the scheduled departure time. This model assumed optimization using the interval halving method (bisection) to achieve the optimal solution. The modified optimization method has been implemented in the original model of railway traffic organization. It considers the optimal use of energy recovered during electrodynamic braking using the energy transmission strategy to the catenary, assuming the cooperation of a train pair and volume of all recovered energy and stop time at the station.

Keywords: rail transport, regenerative braking, optimization of energy recuperation, traffic organization

1. Introduction

The need to optimize the effectiveness of regenerative braking in rail transportation stems from existing transportation problems such as: the need to reduce transportation costs while maintaining acceptable standards, reduce pollutant emissions while maintaining required capacity and increasing demand for transportation services. In addition to the rationalization of running trains [12], it is necessary to introduce modern and ecological technologies as described in documents published by competent national institutions and the European Union [19].

One of the ways to reduce the energy consumption of rail transportation, and therefore lower its costs (including environmental ones), is the use of regenerative technology to recover some of the electricity during electrodynamic braking. The energy recovered in this way can be re-used and thus help improve the energy balance of not only a single run, but also the entire rail transport system.

Among the ways of using electricity derived from regenerative braking, the following methods are distinguished [8, 23, 24]:

- the ability to use it directly in trains for non-traction needs, such as lighting, air conditioning, etc.,
- storing it in stationary or onboard energy storage devices, and then using at time of increased demand [3, 14],
- transmission of recovered energy back to the national power grid [1, 13, 25],
- the transmission of recovered energy back to the electricity transmission infrastructure, allowing the possibility of its immediate use by another vehicle in the acceleration phase [10, 15, 16] or providing acceptable voltage level.

Each of the above methods has advantages and disadvantages [8, 23]. The direct use of recovered energy, notwithstanding AC to DC conversion costs (the situation presented in the Polish railway power system), does not require incurring additional infrastructure modernization costs. The use of energy from regenerative breaking for non-traction purposes of the vehicle or transfer of recovered energy back to the catenary, assuming the energy cooperation of several vehicles, can be called potentially cost-free methods. The method of using energy for non-traction needs of

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the subject train does not require any additional interference in the technical or organizational process itself. Transfer of recovered energy back to the catenary for use by other trains depends on the coordination and train schedules and traffic.

2. Model of energy regeneration by train cooperation

The model described below assumes that by creating an applicable timetable, it will be possible to transfer energy from regeneration of the braking train (train *B*) via the catenary to another vehicle leaving the station (train *A*). Thus, it will reduce the demand for the energy in the acceleration phase transferred from traction substation (Fig. 1). The amount of energy recovered and used in this way will depend on the time of train departure in the range allowed by the detailed timetable.

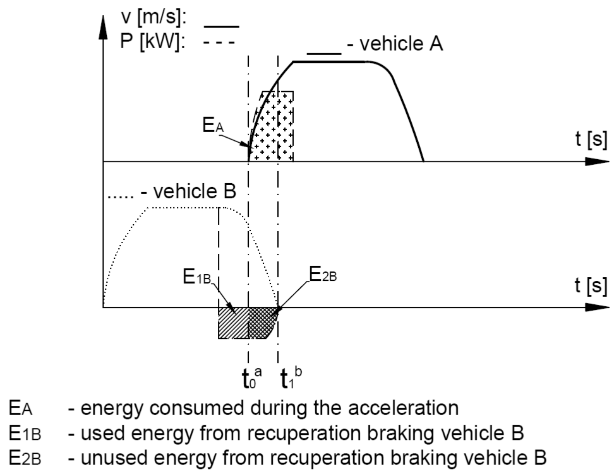


Fig. 1. The use of recovered energy by using control of train arrival time Source: own study based on [20, 21, 24]

The recovery and use of energy from electrodynamic braking requires the cooperation of at least a pair of vehicles, and is mainly possible within stations and stops, where decelerating and accelerating trains are the most frequent processes [9, 10]. The ideal situation would be if the acceleration and deceleration of a pair of vehicles commuting in opposite directions took place at each station or stop of a given railway line in a short period of time (Fig. 2). Unfortunately, in practice this assumption is very demanding and can only be introduced in perfectly functioning (with minimal delays) metro lines. Therefore, this example of a railway line analyzed here assumes the possibility of cooperating only at a few stations and stops.

Although the basics of rail vehicle driving modeling have been widely described in the literature by Podoski et al. [17, 18], nevertheless the computational model proposed there is constantly being modified and refined by various scientists, depending on the goal that needs to be achieved. The model of the theoretical run itself largely comes down to the solution and development of the rail vehicle move equation (Newton's equation), where a train is treated as a material point with mass m. It can be formulated as follows [20, 21, 22, 24]:

$$\begin{cases} km\dfrac{\mathrm{d}v}{\mathrm{d}t} = u(t) - R_s(v) - R_g(x), \\ \dfrac{\mathrm{d}x}{\mathrm{d}t} = v(t), \end{cases} \tag{1}$$

where force $u(t)$ acting on the vehicle is traction force $u = F(t)$ or braking force $u = -F(t)$, depending on the movement phase and k takes into account the moments of inertia of rotating masses. The purpose of these calculations is to determine the parameters of a moving train (e.g., the volume drawn from the catenary, power demand needed to cover the route, etc.), depending on time or distance traveled at the given traction characteristic of the vehicle and with known route geometric parameters.

Resistance of movement $R_s(v)$ is mainly dependent on the aerodynamic forces and wheel – rail interac-

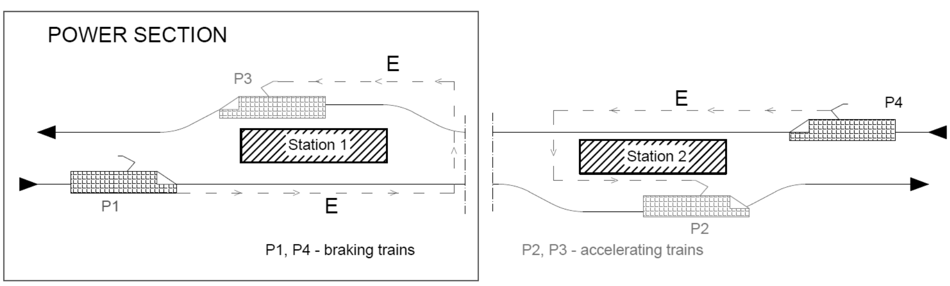


Fig. 2. Scheme of perfect coordination on suburban railway lines or on the subway between braking and starting trains; Source: own study based on [10]

tions. Their dependence on the speed is usually described by a quadratic function [7, 20, 21]:

$$R_s\left(v\right)=k_0+k_1v+k_2v^2\,, \tag{2}$$

where coefficients k_0, k_1, k_2 are constants related to the construction of the rolling stock: with mass and parameters describing the interaction of the wheel with the rail.

The movement resistance $R_g(x) = mgp(x)$ depends on the inclination $p(x)$ of the railway line, which changes with the current position of the train x along the track. This inclination is defined as $p = \Delta h\ /\ l$ where Δh is the difference in height between two track points that are separated by l distance, usually expressed in promiles. Then, the equation takes the form: $R_g(x) = mgp(x) / 1000$.

In addition, if at time t the power consumed by the vehicles during the acceleration phase equals $P_a(t)$ and the power generated during the electrodynamic braking is indicated as $P_{b,r}(t,s)$, and velocity of vehicles A and B are denoted as: $v_a = v_a(t)$ and $v_b = v_b(t)$, the magnitude of $P_a = F(t)v_a(t)$ and $P_{b,r}(t,s) = \varphi(s)B(t)v_b(t)$ can be determined directly from the equation of motion [24]:

$$\begin{cases} P_a\left(t\right)=\left[\dfrac{1}{2}mv_a^2(t+\Delta t)-\dfrac{1}{2}mv_a^2(t)+\Delta E_{loss,a}\right]/\,\Delta t\,, \\ P_{b,r}\left(t,s\right)=\varphi(s)\left[\dfrac{1}{2}mv_b^2(t)-\dfrac{1}{2}mv_b^2(t+\Delta t)-\Delta E_{loss,b}\right]/\,\Delta t\,, \end{cases} \tag{3}$$

where Δt is a short segment of time t, $\varphi(s)$ is a parameter with values ranging between 0 and 1, which depends on the efficiency of energy transfer depending on the distance s between the cooperating trains, and $\Delta E_{loss,a}$, $\Delta E_{loss,b}$ are energy losses incurred to overcome resistance to movement.

In the presented optimization model [20, 21, 24], only the actual energy consumption during the journey is minimized, what takes the form of:

$$E_p = E - E_r, \tag{4}$$

where $E = E_a + E_b$ is the sum of the traction energy consumed by vehicles A and B, and E_r is the part of the energy recovered during braking of the vehicle B and is expressed by:

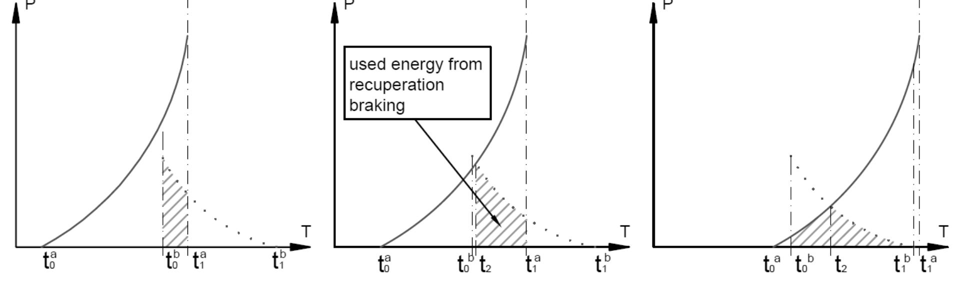
$$E_r=\int_0^T \min\{P_a\left(t\right),P_{b,r}\left(t,s\right)\}\mathrm{d}t\,. \tag{5}$$

For each of these vehicles, the energy consumed during the journey in the time interval $[0,T]$ is expressed by the integral of the power related to the traction force:

$$E_n=\int_0^T \max\left\{u\left(t\right),0\right\}v\left(t\right)\mathrm{d}t=\int_0^T\frac{u\left(t\right)+\left|u\left(t\right)\right|}{2}v\left(t\right)\mathrm{d}t\quad(n=a,b). \tag{6}$$

The specificity of the optimization process takes into account the dependencies between the different departure times from the station and the overlap of decelerating and accelerating rail vehicles as shown in Fig. 3. In the cases considered below, one train (A) is in the acceleration phase and the other train (B) in the braking phase. The start time and end time of the braking (t_0^b and t_1^b) of the train B are the same in all three cases, while the starting time t_0^a of the acceleration phase for the train A changes. It can be delayed up to the limit t_{0gr}^a (latest t_0^a) to allow for a scheduled arrival at the next station.

As the time of departure of train $A(t_0^a)$ increasingly overlaps with the time of breaking of train B, the energy recovered during the braking of the train B



t_0^a - start of acceleration phase
t_0^b - start of braking phase
t_2 - same time of consumption and return energy to catenary
—— volume of power used at a given moment of acceleration
········ volume of power generated at a given moment of braking
t_1^a - end of acceleration phase
t_1^b - end of braking phase

Fig. 3. The use of recovered energy based on control of train departure time; Source: own study based on [20, 21, 24]

as used by the train A, can be shown in the following equations [20, 21, 24]:

– in case 1.:

$$E_r = \int_{t_0^b}^{t_1^a} P_{b,r}\left(t,s\right)\mathrm{d}t\,, \tag{7}$$

– in case 2.:

$$E_r = \int_{t_0^b}^{t_2} P_a\left(t,t_0^a\right)\mathrm{d}t + \int_{t_2}^{t_1^a} P_{b,r}\left(t,s\right)\mathrm{d}t, \tag{8}$$

– in case 3.:

$$E_r = \int_{t_0^b}^{t_2} P_a\left(t-t_0\right)\mathrm{d}t + \int_{t_2}^{t_1^b} P_{b,r}\left(t,s\right)\mathrm{d}t. \tag{9}$$

After analyzing how the use of regenerative energy E_r changes depending on the departure time t_0^a (increases in cases 1. and 2. or decreases in cases 2. and 3.), it is evident that the model can be described by an unimodal function. Further, it should be noted that according to the following relations as presented in the works [20, 21, 24]:

$$\begin{cases} \dfrac{dE_p}{dt_0^a} < 0, \ \text{gdy } t_0^a < t^{**}, \\ \dfrac{dE_p}{dt_0^a} = 0, \ \text{gdy } t_0^a = t^{**}, \\ \dfrac{dE_p}{dt_0^a} > 0, \ \text{gdy } t_0^a > t^{**}, \end{cases} \tag{10}$$

The actual traction energy consumption E_p will decrease in the first stage, and then increase with the delay in the departure time of the train from the station. This implies that the minimum actual traction energy consumption E_p occurs for the departure time $t_0^a = t^{**}$. This time in turn can be determined using the equal division method (bisection) [4, 5] by solving the non-linear equation:

$$\mathrm{d}E_p / \mathrm{d}t_0^a = 0\,. \tag{11}$$

The solution to this equation is based on the assumption of the earliest and possibly latest time of departure of the train from the station and determination of the gradient of actual traction energy consumption for these two departure times. If both the solved values are positive, the earliest possible departure time from the station is the optimal solution. In any other case, we determine the optimal value t_0^a by gradually narrowing the range of train departure times and analogically we look for the optimal solution using the bisection method applied to the function $\mathrm{d}E_p / \mathrm{d}t_0^a$ in which both gradients have positive values [24].

3. Modification of the model

At the outset, it should be assumed that the traction network is supplied with constant voltage (e.g. 3 kV DC, which is a typical supply voltage on Polish railway lines), and the rolling stock and infrastructure will make it possible to use regenerative braking technology with the transfer of recovered energy to the catenary [11]. In the following model, in contrast to the original model described in the previous section, a variable controlling the theoretical run in the form of braking end time (t_{KH}^B) was proposed. This value is equivalent to the actual time of arriving train B at the station or stop i.e., $t_{KH}^B = T_{RP}^B$. This situation is illustrated in Figure 4.

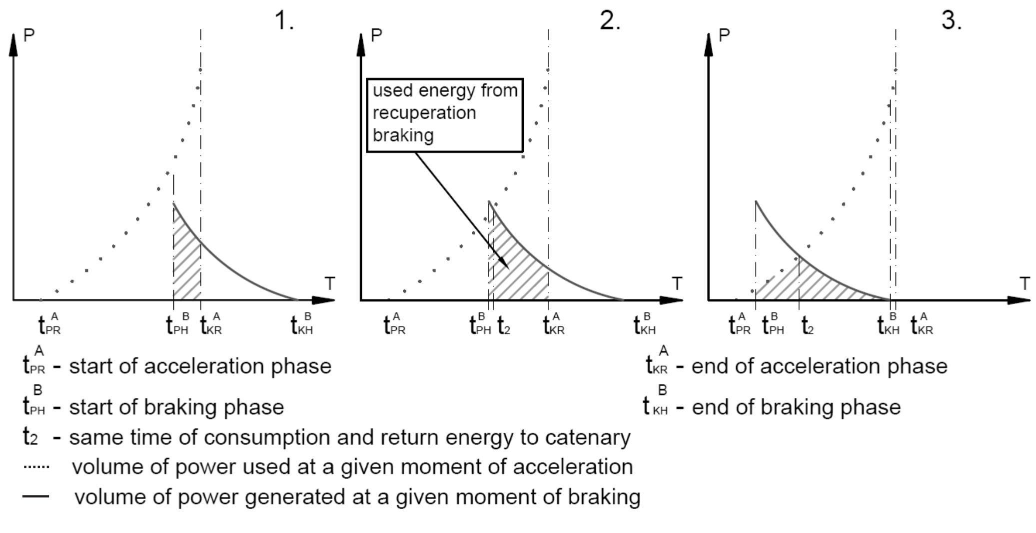


Fig. 4. Changes of the use of recovered energy by using control of train arrival time [own study]

Thus, we have a situation where the start and end times of the acceleration phase for vehicle *A* are constant ($t^A_{PR} = const.$ and $t^A_{KR} = const.$), and that means the characteristics of movement of *A* train does not change in terms of acceleration and the amount of energy required for the acceleration phase. On the other hand, the start and stop times of the braking phase for the train *B* (t^B_{PH} and t^B_{KH}) shall be shifted, with the earliest braking instance of the train *B* being earlier than the starting time of train *A* ($t^B_{KH} \geq t^A_{PR}$) and the latest time of braking of the train *B* cannot occur later than the moment the train *A* is started ($t^B_{PH} \leq t^A_{KR}$). Further, as stated before, the actual time of arrival of train *B* at the station T^B_{RP} must be within the range between the earliest and the latest time of arrival of a given train to a station pursuant to the official detailed timetable: $T^B_{RP} \in \left\langle T^B_{PP}; T^B_{WP} \right\rangle$.

Further, other basic assumptions of the theoretical run model (1) – (3) are unchanged. Additionally, knowing that the traction network is supplied with constant voltage, the DC power can be expressed by the product of the voltage *U* and the current *I*:

$$P(t) = \eta U I(t), \tag{12}$$

where η is an efficiency of engine (η_E) or efficiency or regenerative braking (η_B) and $I(t)$ is the current drawn or generated by train.

With this substitution we find:

$$\begin{cases} \eta_E U I_A(t) = \left[\frac{1}{2} m v_a^2(t+\Delta t) - \frac{1}{2} m v_a^2(t) + \Delta E_{loss,a} \right] / \Delta t, \\ \eta_B U I_B(t,s) = \varphi(s) \left[\frac{1}{2} m v_b^2(t) - \frac{1}{2} m v_b^2(t+\Delta t) - \Delta E_{loss,b} \right] / \Delta t, \end{cases} \tag{13}$$

where the energy necessary to overcome the resistance of movement is expressed by [22]:

$$\begin{aligned} \Delta E_{loss,a} &= \left[R_{s,a}(v_a) + R_{g,a}(x_a) \right] \Delta x_a, \\ \Delta E_{loss,b} &= \left[R_{s,b}(v_b) + R_{g,b}(x_b) \right] \Delta x_b, \end{aligned} \tag{14}$$

where $\Delta x_a = v_a \Delta t$ and $\Delta x_b = v_b \Delta t$ are the lengths of the track sections passed by: vehicle *A* being in the acceleration phase and vehicle *B* being in the braking phase.

The current absorbed / induced can be determined by knowing the force needed to overcome the resistance of movement $F(t)$, the current train speed $v(t)$ and the efficiency of the engine / generator η:

$$I = F(t) \cdot \frac{v(t)}{U \cdot \eta}. \tag{15}$$

Force $F(t)$ for the train's acceleration phase can be determined from [17, 18]:

$$F_{A1} = k \cdot \mathrm{m} \cdot a_A. \tag{16}$$

for acceleration with constant force, where *k* is the factor of rotating masses, *m* is weight of vehicle, a_A is maximum acceleration, or for acceleration with constant maximum power:

$$F_{A2} = \frac{P_E \cdot \eta_E}{v_A(t)}, \tag{17}$$

where P_E is power of all vehicle engines and $v_A(t)$ is the characteristic speed calculated from the condition $F_{A1} = F_{A2}$.

Similarly $F(t)$ for the breaking train can be described as:

$$F_{B1} = k \cdot \mathrm{m} \cdot a_B, \tag{18}$$

$$F_{B2} = \frac{P_E \cdot \eta_E}{v_B(t)}, \tag{19}$$

where a_B is maximum deceleration and $v_B(t)$ is the characteristic speed corresponding to $F_{B1} = F_{B2}$.

Since the highest demand for power and electricity occurs during the acceleration phase, these needs can be reduced by appropriate energy management from the regenerative braking of another train. The electricity demand in this case will be equal to the energy balance needed to accelerate less the energy recovered from electrodynamic braking [18, 19, 24] of the breaking train. According to the proposed arrival time control approach, there is no need to optimize the entire trip in order to achieve relatively measurable benefits in the energy balance, but only the part of the trip when the braking and acceleration take place needs optimization. Therefore, the main component of the objective function is proposed to be:

$$E_P = E - E_{RW} \rightarrow \min, \tag{20}$$

where E_P is the actual energy value consumed during the acceleration phase of the vehicle *A*, *E* is the all energy required to perform the acceleration, and E_{RW} is the energy recovered during the braking of the electrodynamic vehicle *B* and used in the cooperation process of both trains. Referring to (12), individual energy values can be determined as:

$$E = U \int_{T_{PR}}^{T_{KR}} I(t) dt, \tag{21}$$

$$E_{RW} = U \int_{T_{PR}}^{T_{KH}} \min\{I_A(t), I_B(t,s)\} dt, \quad (22)$$

where $I_A(t)$ is the current drawn by the accelerating train (A) and $I_B(t,s)$ is the current enerated by the braking train (B).

It was also found that there are other criteria that should be considered using a multi-criteria optimization, including:

$$E_{RO} = U \int_{T_{PH}}^{T_{KH}} I_B(t) dt \to \max, \quad (23)$$

where E_{RO} is the entire energy recovered during the braking of train B that can be used in a different way other than direct transmission to the catenary (e.g. additional energy storage) and:

$$T_P = T_O^B - T_{RP} \to \min, \quad (24)$$

where T_P is the stop time, which should be as minimal as possible due to the station's capacity, T_O^B is the scheduled time of departure of the train B, while T_{RP} is its actual arrival time.

Taking the above into consideration, we obtain the following global objective function:

$$F(T_O^B, T_{RP}) = w_1 E_P - w_2 E_{RO} + w_3 T_P \to \min, \quad (25)$$

where w_1, w_2, w_3 are the validity of the individual member functions reflecting the importance of the function of the criteria in the existing railway network conditions. This function is minimized, therefore, partial functions that are also minimized are written with a positive sign (E_P,T_P) and the maximized partial function (E_{RO}) is written with a negative sign.

By using presented model optimized volume of recuperated and used energy of cooperating trains for the Gdańsk Żabianka AWFiS railway station on the railway line no. 250 Gdańsk Główny – Rumia (Tab. 1). As a result of optimization by Firefly Algorithm, it has been proved that within 24 hours, the volume of electric energy of the cooperating train (at the analyzed station) can be recovered E_{RO} = 97.44 kWh and E_{RW} = 65.44 kWh can be used in the process of energy cooperation of trains. For total volume of electricity demand of cooperating train E_P = 162.10 kWh (on the analyzed section: Gdańsk Oliwa – Gdańsk Żabianka AWFiS – Sopot Wyścigi) energy used directly from recuperation accounts for over 40% volume of energy. That means the real demand for electric traction energy volume is 96.66 kWh. This result is close to the levels of energy volume possible to be used in trains energy cooperation presented in [2] or [6].

Table 1

Results of optimization using the firefly algorithm for a train stop Gdańsk Żabianka AWF

	E_P [kWh]	E_{RO} [kWh]	E_{RW} [kWh]
	15.3999	9.7495	5.8728
	15.9565	9.5664	6.9408
	18.8695	10.0022	7.1727
	15.3999	9.7495	6.3806
	17.0331	9.2331	6.5425
	15.4019	9.7509	6.3814
	17.0616	9.3467	6.5860
	15.3885	9.7153	6.3669
	16.1888	10.5684	6.8135
	15.4021	9.7543	6.3826
Total:	**162.10**	**97.44**	**65.44**

Source: own study

4. Conclusions

In conclusion, we note that there are some models that already outline ways to optimize the use of energy from regenerative breaking using transmission to the catenary and mutual cooperation of several vehicles – e.g. by controlling the time of departure of the train. However, there are still many other unrecognized possibilities to increase the efficiency of regenerative braking, such as energy optimization with the use of a reserve of passage time included in the timetable by controlling the arrival time. These methods, although similar, differ in the way trains run and consume energy. In the first case (control of train departure time) along with its gradual departure delay, the energy necessary to pass the next section is increased, e.g., since higher speeds need to be obtained. In the second case (control of time of arriving train at the station), with the arrival delay, the energy demand decreases, e.g., due to extended run without power consumption, need for lower speeds or possibility of maximum braking deceleration.

Existing models can still be modified and developed adapting them to the set of requirements, existing (changing) conditions and needs – e.g., by developing and explaining the record, introducing the ability to edit basic data or giving up some of the calculations by replacing them with other dependencies potentially shortening the whole calculation and analysis process.

In the context of optimization, it should be emphasized that the organization of train traffic (or other rail transport systems) cannot be modified only in terms of optimizing energy consumption or regenerative breaking efficiency. Due to the superior criteria, such as a specific travel time, stoppage time, capacity of railway lines and stations and demand side requirements, it is possible to reorganize the traffic but only in a narrow range. This was reflected both in the assumptions of the model $\left(T_{RP}^{B} \in \left\langle T_{PP}^{B}; T_{WP}^{B} \right\rangle\right)$ and in the minimized global objective function.

The use of the proposed model along with the optimization in the analysis of a single railway stop showed the possibility of saving over 40 % of traction electricity.

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