

# Determination of Inductive Parameters of the Uncompensated DC Machines Taking Into Account the Reaction of the Armature

Anton DRUBETSKY<sup>1</sup>

## Summary

In modeling the electrical machine, for calculation transient states, it is necessary to determine the inductances of the coils. This problem can be solved in different ways. There know the design parameters of the machine, being available to it basic magnetizing curve or directly from the experiment. As a rule we don't have the calculated data of the specific engine when solving a modeling problem. Also we need expression allows to obtain the value of the inductive parameters for each possible value of the currents flowing in the motor coils for studding different modes of operation. This expression can be obtained using the magnetizing curve of the machine when open-circuit operation. It is known that when working uncompensated machine a significant impact on the magnitude of the magnetic flux provided by the armature reaction which in turn has an impact on its inductance. In this case, in the determination of inductive parameters we have to take into account the effect of the armature reaction. The method of determining the inductive parameters of the uncompensated traction electric motor taking into account back induction is described in this article. This method allows to obtain analytical expressions for the inductive parameters that can be directly used for simulation of transient electromagnetic processes in the case that linearization of these parameters is unacceptable is making a gross error in the calculations. The influence of eddy currents in the work is not taken into account.

**Keywords:** DC machine, traction motor, armature reaction, magnetizing curve, flux per pole, simulation

## 1. Introduction

As you know, when the direct-current machine under load, by another name when the armature current is different from zero, there is an effect called armature reaction of direct-current machine [7–10, 16]. It arises because of the imposition of the magnetic field of the armature in the magnetic field of main poles, resulting in distortion of the magnetic field under the main poles. In addition to distorting operation, armature reaction is a decrease in the magnetic flux of the main pole (main magnetic flux). This is especially true at high armature currents and deep attenuation of the excitation [9]. It is obvious that the modeling of electromagnetic processes in electrical machines, and in particular different regimes at weak excitation, there is a need to take into account the demagnetizing effect of armature reaction.

## 2. Purpose

The purpose of this work is the analysis of the analytical accounting treatment of the armature reaction

and the development of the method of constructing the magnetization curve taking into account the back induction of armature reaction for uncompensated direct-current machine.

## 3. The justification of methods of calculation

At the moment there are several methods based on the armature reaction on the main magnetic flux. A general approach to account for the influence of the magnetomotive force (MMF) of armature reaction on the main magnetic flux is described in the fundamental works on electric machines [8, 10, 16]. Also, in these works, there are some general methods to account for this effect. In [8] indicated that for the calculation of stationary and transient processes on computer characteristics of magnetization can be conveniently represented in the form of approximations, however, are not given clear guidance for the analytical calculation. In [9] on the theory of traction electric machines, the method in which the reduction of the magnetic flux is calculated by introducing the demagnetizing factor

<sup>1</sup> Postgraduate student; Dnipropetrovsk National University of Railway Transport Named after Academician V. Lazaryan; e-mail: drubetskiy@mail.ru.

which is determined experimentally and can be applied for different types of traction engines by entering the relevant amendments. As stated in [13], this method is most common in engineering practice. In the same paper, which is devoted to the design of the electric traction machines there are two methods of calculation. In reviewing the above techniques, it can be concluded that the methodology described in [9] is essentially graphic, a technique described in [8, 10, 13, 16] is graphic. These methods are well applied in engineering practice in designing electric cars, but for modeling, you need an analytical expression, which could determine the magnetic flux for all possible changes of MMF and MMF excitation of the armature reaction. In [3], devoted to transient processes in the DC micro machines, presents an analytical method considering the influence of MMF of the reaction anchor. Therefore, to development analytical expressions that describe the main magnetic flux, we use an analytical method [3].

#### 4. Justification of the choice of initial data for the calculation

In the General course of electrical machines [8, 10, 16], the MMF separation of the quadrature-axis armature reaction  $F_{aq}$  and direct axis armature reaction  $F_{ad}$ . This principle of separation is physically justified and adopted for all kinds of direct-current machines. According to this splitting, the direct axis armature reaction  $F_{ad}$  occurs due to a shift of the brushes from the geometric neutral line (on the pole axis). In this article, we assume that the brushes set strictly at geometric neutral line, as a result  $F_{ad} = 0$ .

To determine the magnetic flux under load we must have the characteristic of magnetization of the machine open-circuit operation  $\Phi_d(F_{FW})$  (hereinafter is the magnetization characteristic) or mutual characteristic  $B_\delta(F_{\delta Z})$ , where  $F_{FW}$  – MMF of field magnetizing coil,  $F_{\delta Z}$  – the sum of the drop in magnetic voltage in the gap and armature projections.

In the preliminary calculation revealed that to obtain analytical expressions for the magnetic flux on the mutual characteristic it is necessary to perform some approximations:

- mutual characteristic;
- magnetizing curve tooth layer of the machine and the armature heelpiece;
- magnetizing curve of cast steel machine frame and main poles.

This is in turn introduces additional uncertainty into the calculations.

Therefore, to obtain analytical expressions for the main magnetic flux, we use the characteristic magnetization of the machine  $\Phi_d(F_{FW})$ .

## 5. Development analytical expressions according to the chosen methodology

### 5.1. Development expressions for the main magnetic flux

We can use one of the existing mathematical models of curves of magnetization to approximate the characteristics of magnetization of the machine [2, 12, 15]. Also we can use the methodology described in [1] in the case of known coefficient of magnetic saturation of the studied machine. Since, in General, for the studied machines the coefficient of magnetic saturation is unknown, we use one of the mathematical models described in [12]. As approximate expression, the arc tangent function is taken as one of the functions that most accurately describes the magnetization curve [12]. This feature is known in the literature as the formula of Dreyfus. For the characterization of magnetization it has the form:

$$\Phi_d(F_{FW}) = p_1 \arctg(p_2 F_{FW}) + p_3 F_{FW}, \quad (1)$$

where  $p_1, p_2, p_3$  – the coefficients of approximation;  $F_{FW}$  – MMF of field magnetizing coil.

Under load, the main magnetic flux also depends on the MMF reaction of the armature current within the estimated polar pitch  $b_\delta$

$$F_{aq}^* = \frac{1}{2} b_\delta \frac{N}{2a\pi D_a} i_a \quad (2)$$

where  $i_a$  – the armature current, A. Here, in the future, it is assumed that the armature current, in General, depends on the time, so it is denoted by a small letter;  $b_\delta$  – calculated polar pitch;  $N$  – the number of conductors of the armature winding;  $D_a$  – armature diameter, m;  $a$  – the number of pairs of parallel branches of the armature winding.

Also, high capacity machines, which include traction motors, a significant effect on a primary magnetic flux having a switching MMF, which occurs due to currents in the short-circuited sections. The direction of switching MMF depends on the nature of switching. If switching is speed-up, that switching MMF is directed oppositely to the MMF of main poles and has a demagnetizing effect. If switching is slow down, that switching MMF. At the traction motors the commutation is accelerated due to the presence of additional poles. Given the above resultant MMF, which creates

the main magnetic flux is the difference of the MMF of the field magnetizing coil and the switching MMF. Switching MMF to a wide range of changes in the armature current has the form [14]

$$F_k = 0,02i_a^2. \quad (3)$$

To simplify further expressions, we assume that in  $F_6$  is already taken into account the effect of switching MMF. Thus, you have not actually MMF of field magnetizing coil, and the difference between the MMF of the coils of excitation and commutation of MMF.

It should be noted that the accounting effect of  $F_k$  is better done at the stage of verification when the obtained expression for the main magnetic flux and have the opportunity to verify the characteristic of the magnetization given in the literature or obtained experimentally. Thus, it is advisable to calculate the effect of  $F_k$ , but only for the in the case of large differences between calculation and experience, you can enter into  $F_k$  to the equation for the main magnetic flux.

According to [3], for the main magnetic flux, taking into account the reaction of the armature, we have:

$$\Phi_{dq}(F_{FW}, F_{aq}^*) = \frac{1}{2F_{aq}^*} \int_{F_{FW}-F_{aq}^*}^{F_{FW}+F_{aq}^*} \Phi_d(F) dF. \quad (4)$$

By substituting the expression (1) in the expression (2) designating and using  $F_1 = F_{FW} - F_{aq}^*$  and  $F_2 = F_{FW} + F_{aq}^*$ , we obtain an expression for the main magnetic flux in the machine running under load:

$$\begin{aligned} \Phi_{dq}(F_{FW}, F_{aq}^*) &= \frac{P_1}{2F_{aq}^*} (F_2 \arctg(p_2 F_2) - F_1 \arctg(p_2 F_1)) - \\ &+ \frac{1}{2p_2} \ln \left( \frac{1+p_2^2 F_2^2}{1+p_2^2 F_1^2} \right) + p_3 F_{FW}. \end{aligned} \quad (5)$$

The expression (4) gives the possibility to determine the true magnetic flux in the machine under load, with only the characteristic of the magnetizing machine (an open-circuit operation characteristic).

## 5.2. Obtaining general expressions for inductive parameters of electric motor

Having the expression for the primary magnetic flux at any load, there is a possibility of determination of inductive parameters of the electric machine in dynamic mode, for example when working it in quasi-stationary mode when powered by a pulsed voltage source. To retrieve the data dependencies it is neces-

sary to write the equation of electromagnetic state of uncompensated traction motors of series excitation:

$$L_{FW\sigma} \frac{d(i_a \beta)}{dt} + (L_{a\sigma} + L_{ap\sigma} + L_{ap}) \frac{di_a}{dt} + \frac{d\Psi_{FW}}{dt} + \frac{d\Psi_{aq}}{dt} + i_{FW}(t)R_{FW} + i_a(t)(R_a + R_{ap}) + e(\Phi_{dq}, \omega) + \Delta U_b = u(t), \quad (6)$$

where  $i_a(t)$  – current of armature winding;

$i_{FW}(t) = i_a(t)\beta$  – current of field winding coil, where  $\beta$  – attenuation degree of excitation field. For direct-current machines  $\beta \in [\beta_{\min}; 1]$ , for intermittent-cycle engine  $\beta \in [\beta_{\min}; (\beta_{\max} < 1)]$ ;  $\beta_{\max}$  always less than unity due to the presence of permanently enabled shunt resistance. In the particular case for direct-current machines, when  $\beta = 1$ ,  $i_{FW}(t) = i_a(t)$ ;

$u(t)$  – the voltage of the power supply. When the motor supply of direct current source  $u(t) = U_s = \text{const}$ , and when the motor supply of intermittent-cycle source with period  $T = 1/f$  and pulse time  $t_i$ :

$$u(t) = \begin{cases} U_s & 0 \leq t \leq t_i; \\ 0 & t_i < t \leq T. \end{cases} \quad (7)$$

$e(\Phi_{dq}, \omega)$  – the EMF of rotation (counter electromotive force) dependent on the main magnetic flux and the rate of phase change of rotation of the armature,  $e(\Phi, \omega) = c\Phi_{dq}(F_{FW}, F_{aq}^*)\omega$ ;

$R_a, R_{ap}, R_{FW}$  – active resistance of windings of the armature, additional poles and field magnetizing coil;  $L_{a\sigma}, L_{ap\sigma}, L_{FW\sigma}$  – leakage inductance of the windings of the armature, additional poles and field magnetizing coil;

$L_{ap}$  – the inductance of the additional poles, as the magnetic system additional pole is made of unsaturated, then it can be considered a constant inductance in the operating range of motor currents;

$\Delta U_b$  – the voltage drop on the brushes;

$\Psi_{aq}, \Psi_{FW}$  – flux linkage from the main magnetic flux of the windings of the armature and excitation for all poles [7]:

$$\Psi_{FW} = 2pw_{FW}\Phi_{dq}(F_{FW}, F_{aq}^*); \quad (8)$$

$$\Psi_{aq} = \frac{2pw_a'}{2F_{aq}^*} \int_{F_{FW}-F_{aq}^*}^{F_{FW}+F_{aq}^*} (F - F_{FW})\Phi_d(F_{FW}) dF. \quad (9)$$

where  $p$  – the number of pairs of poles;

$w_{FW}$  – the number of turns of the excitation winding;

$w_a'$  – the number of turns of armature windings per pole (one polar pitch  $\tau$ ):

$$w_a' = \frac{N}{8ap}.$$

In accordance with [3] expressions  $d\Psi_{aq}/dt$  and  $d\Psi_{FW}/dt$  can be represented in the form:

$$\frac{d\Psi_{aq}}{dt} = \frac{\partial\Psi_{aq}}{\partial i_a} \frac{di_a}{dt} + \frac{\partial\Psi_{aq}}{\partial i_{FW}} \frac{di_{FW}}{dt}; \quad (10)$$

$$\frac{d\Psi_{FW}}{dt} = \frac{\partial\Psi_{FW}}{\partial i_a} \frac{di_a}{dt} + \frac{\partial\Psi_{FW}}{\partial i_{FW}} \frac{di_{FW}}{dt}. \quad (11)$$

According to the definition of inductance, and mutual inductance [11] partial derivatives-current in expressions (10) and (11) can be denoted:

$$\frac{\partial\Psi_{aq}}{\partial i_a} = L_{aq}(F_{FW}, F_{aq}^*); \quad (12)$$

$$\frac{\partial\Psi_{aq}}{\partial i_{FW}} = M_{a-FW}(F_{FW}, F_{aq}^*); \quad (13)$$

$$\frac{\partial\Psi_{FW}}{\partial i_a} = M_{FW-a}(F_{FW}, F_{aq}^*); \quad (14)$$

$$\frac{\partial\Psi_{FW}}{\partial i_{FW}} = L_{FW}(F_{FW}, F_{aq}^*) \quad (15)$$

where  $L_{aq}(F_{FW}, F_{aq}^*)$ ,  $L_{FW}(F_{FW}, F_{aq}^*)$  – the inductance of the armature windings and field magnetizing coil;  $M_{a-FW}(F_{FW}, F_{aq}^*)$ ,  $M_{FW-a}(F_{FW}, F_{aq}^*)$  – mutual induction between armature and magnetizing coil, magnetizing coil and armature.

It is obvious that due to the nonlinear dependence of the main magnetic flux from the MMF of the field winding and armature reaction  $\Phi_{dq}(F_{FW}, F_{aq}^*)$ , induction machine parameters defined in expressions (12–15) are also nonlinearly dependent on these MMF.

### 5.3. Determination of inductive parameters of the motor given the reaction of the armature

As is known, the flux linkage coil with the current can be determined not only as the product of the current

$$L_{aq}(F_{FW}, F_{aq}^*) = 2pw_a'^2 \left( \frac{p_1}{p_2 F_{aq}^{*2}} \left( 1 + \frac{p_2^2 F_{FW}^2 - 1}{2p_2 F_{aq}^*} (\arctg(p_2 F_2) - \arctg(p_2 F_1)) - \frac{F_{FW}}{2F_{aq}^*} \ln \left( \frac{1+p_2^2 F_2^2}{1+p_2^2 F_1^2} \right) + \frac{p_3}{3} \right) \right); \quad (20)$$

$$M_{FW-a}(F_{FW}, F_{aq}^*) = M_{a-FW}(F_{FW}, F_{aq}^*) = -2pw_{FW} w_a' \frac{p_1}{2F_{aq}^{*2}} \left( F_{FW} (\arctg(p_2 F_2) - \arctg(p_2 F_1)) - \frac{1}{2p_2} \ln \left( \frac{1+p_2^2 F_2^2}{1+p_2^2 F_1^2} \right) \right); \quad (21)$$

$$L_{FW}(F_{FW}, F_{aq}^*) = 2pw_{FW}^2 \left( \frac{p_1}{2F_{aq}^*} (\arctg(p_2 F_2) - \arctg(p_2 F_1)) + p_3 \right). \quad (22)$$

in the inductance, and as the product of the number of turns in the magnetic flux linked with the coil [4]. This approach is used in the derivation of expressions (8) and (9). As in the expression (4) as arguments used of MMF, the inductive parameters are more convenient using the derivative of flux linkage for MMF of the armature and field magnetizing coil. For this expression (12–15) multiply and divide by the number of turns of the respective windings, according to MMF which the derivative is taken. Given this, expressions for the inductive parameters of the motor (12–15) will take the form of:

$$L_{aq}(F_{FW}, F_{aq}^*) = 2pw_a'^2 \left( \frac{\partial\Phi_{dq}(F_{FW}, F_{aq}^*)}{\partial F_{FW}} \frac{\int_{F_{FW}-F_{aq}^*}^{F_{FW}+F_{aq}^*} (F-F_{FW}) \Phi_d(F) dF}{F_{aq}^{*3}} \right); \quad (16)$$

$$M_{a-FW}(F_{FW}, F_{aq}^*) = \frac{2pw_a' w_{FW}}{2F_{aq}^{*2}} \left( \frac{\partial \left( \int_{F_{FW}-F_{aq}^*}^{F_{FW}+F_{aq}^*} (F-F_{FW}) \Phi_d(F) dF \right)}{\partial F_{aq}^*} \right); \quad (17)$$

$$M_{FW-a}(F_{FW}, F_{aq}^*) = 2pw_a' w_{FW} \frac{\partial\Phi_{dq}(F_{FW}, F_{aq}^*)}{\partial F_{aq}^*}; \quad (18)$$

$$L_{FW}(F_{FW}, F_{aq}^*) = 2pw_{FW}^2 \frac{\partial\Phi_{dq}(F_{FW}, F_{aq}^*)}{\partial F_{FW}}. \quad (19)$$

By substituting the expression (4) in expression (16–19) and considering that  $M_{a-FW}(F_{FW}, F_{aq}^*) = M_{FW-a}(F_{FW}, F_{aq}^*)$  obtain the final expression for the inductive parameters of the motor [3]:

## 6. Calculated and experimental study of induction parameters of motor taking into account the reaction of the armature

As mentioned in the previous section, for the determination of inductive parameters of the machine we need to have the characteristics of magnetization of the electrical machine. The characteristic of magnetization can be determined by characteristics of open-circuit operation. Figure 1 shows the circuit for rating the load-voltage characteristics.

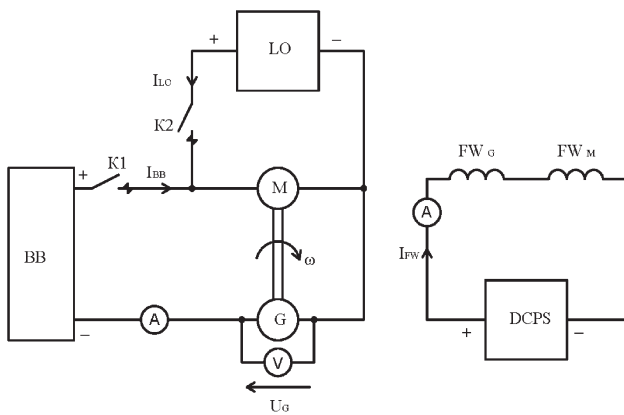


Fig. 1. The circuit for rating the load-voltage characteristics

In Fig. 1 is shown: BB – balancing booster, LO – linear oscillator, DCPS – DC power supply, K1, K2 – contactors connecting to the scheme respectively balancing booster and linear oscillator, M – armature of machine which working like engine, G – armature of machine which working like generator unit,  $U_g$  – voltage on the armature of generator unit,  $I_{BB}$  – current of balancing booster,  $I_{LO}$  – current of linear oscillator,  $FW_M$ ,  $FW_G$  – field winding of motor and generator unit,  $I_{FW}$  – current of field winding,  $\omega$  – rotation frequency of the studied machines.

The rating of open-circuit operation is made at the switched-off contactor K1 and switched-on contac-

tor K2. Small armature current of the motor M is set by using the linear oscillator. Then, by changing the excitation current from minimum to maximum with the preset step, are rotated the studied machine, the rotation frequency is maintained within the operating range. At each step of change excitation current, a measure of the voltage on the armature of the generator and rotational speed are done. Then, including the contactor K1 is set to required, the armature current of the generator, which is equal to the current of balancing booster, and re-iterates the above measurement. As sources, sources with constant current are prefer. After rating of open-circuit operation, the main magnetic flux is determined by the formula:

$$\Phi_d(F_{FW}) = \frac{U_g}{c \cdot \omega}, \quad (22)$$

When the generator is under load, the main magnetic flux is determined by the formula:

$$\Phi_{dq}(F_{FW}, F_{aq}^*) = \frac{U_g - I_{BB} R_{aG}}{c \cdot \omega} \quad (23)$$

where  $R_{aG}$  – the resistance of the armature of the generator unit.

Open-circuit characteristic of the traction motor RT-51M ( $R_{aG} = 0,056 \text{ Ohm}$ ) and the magnetic flux defined by expression (21), are presented in Table 1.

According to Table 1 was executed approximation of the magnetization curve according to the expression (1):

$$\Phi_d(F_{FW}) = -0,0484434085 \cdot \arctg(-0,0002353001 \cdot F_{FW}) + 0,0000003254 \cdot F_{FW}, \quad (24)$$

Table 2 presents the results of the calculation of the magnetic flux according to the expression (23):

Table 1

$I_{FW}$ , A	64,02	82,69	101,26	129,33	150,91	182,97	209	228,57
$U_g$ , V	321,83	326,03	316,94	317,08	354,1	355,74	353,37	352,33
$\omega$ , rad/s	53,75	46,28	40,73	37,05	39,18	37,23	35,61	34,55
$\Phi_d(F_{FW})$ , Wb	0,04002	0,047087	0,052015	0,057204	0,060408	0,063874	0,066332	0,06816

Table 2

$I_{BB}$ , A	64,02	82,69	101,26	129,33	150,91	182,97	209	228,57
$\Phi_d(F_{FW})$ , Wb, experiment	0,04002	0,047087	0,052015	0,057204	0,060408	0,063874	0,066332	0,06816
$\Phi_d(F_{FW})$ , Wb, approximation	0,040046	0,046571	0,051548	0,057149	0,060413	0,064198	0,066643	0,068222
$\Delta$ , %	0,065	1,096	0,898	0,095	0,007	0,508	0,469	0,091

As can be seen from table 2, the approximation of the arc tangent function gives a good approximation to the true characteristic of magnetization. Graphically the experimental and approximated characteristics of magnetization are presented in Figure 2.

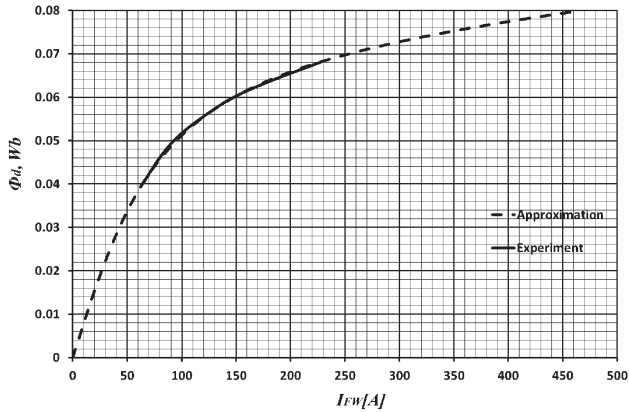


Fig. 2. Approximation of the characteristics of magnetization at open-circuit operation

To check the accuracy of the calculation of the characteristics of magnetization according to the expression (4) or (5) there were several points when the armature current of the generator, different from zero. The results of experiment and calculation, as well as the degree of divergence of the calculated values and experimental data are presented in Table 3.

As can be seen, the difference does not exceed 7%, indicating acceptable accuracy of the calculated expression for the main magnetic flux  $\Phi_{dq}(F_{FW}, F_{aq}^*)$ . Graphically, the results presented in table 3 are shown in Figure 3.

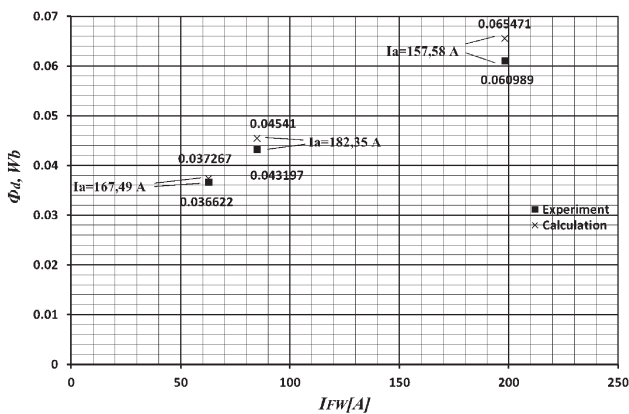


Fig. 3. Calculation and experimental values of the main magnetic flux under load

To determine the validity of the obtained expressions for the main magnetic flux, it is possible to quantify the inductive parameters of the traction motor PT-51M. It is necessary to substitute expression (23) in expression (18–20), taking the same current in the armature and the field winding. The calculation results are presented in Table 4 and Figures 4, 5, 6.

Table 4

I, A	$L_{dq}(F_{FW}, F_{aq}^*)$ , mH	$M_{FW-a}(F_{FW}, F_{aq}^*)$ , mH	$L_{FW}(F_{FW}, F_{aq}^*)$ , mH
0	13.499	-0.004294	216.85
20	12.238	-1.424	196.973
40	9.696	-3.468	155.752
60	7.336	-4.237	116.964
80	5.559	-4.059	87.841
100	4.295	-3.538	67.354
120	3.402	-2.979	53.044
140	2.762	-2.485	42.904
160	2.295	-2.077	35.566
180	1.947	-1.747	30.134
200	1.683	-1.482	26.026
220	1.478	-1.269	22.857
240	1.316	-1.095	20.368
260	1.187	-0.953	18.382
280	1.082	-0.836	16.775
300	0.996	-0.739	15.457
320	0.924	-0.657	14.364
340	0.864	-0.588	13.448
360	0.813	-0.528	12.673
380	0.77	-0.477	12.013
400	0.733	-0.433	11.445
420	0.7	-0.395	10.954
440	0.672	-0.362	10.526
460	0.647	-0.332	10.151

Table 3

$I_{FW}$ , A	$I_{BB}$ , A	$U_g$ , B	$\omega$ , rad/s	$\Phi_{dq}(F_{FW}, F_{aq}^*)$ , Wb, experiment	$\Phi_{dq}(F_{FW}, F_{aq}^*)$ , Wb, calculation	$\Delta$ , %
62,7	167,49	292,33	51,618	0,036622	0,037267	1,73
85,07	182,35	293,44	43,805	0,043197	0,04541	4,87
198,22	157,58	291,39	30,953	0,060989	0,065471	6,85

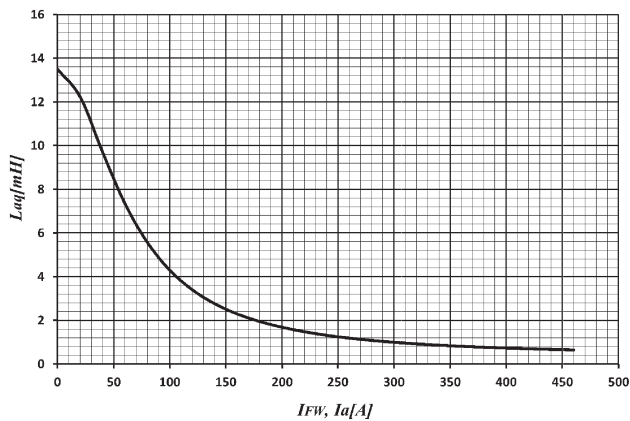


Fig. 4. The dependence of the inductance of the armature winding from the excitation current and armature current

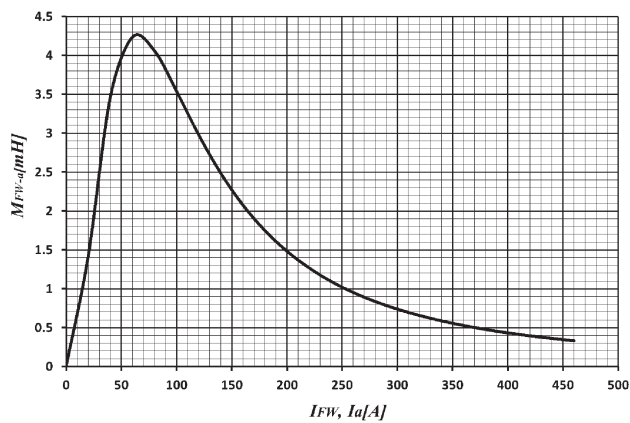


Fig. 5. The dependence of the mutual induction between field windings and armature of the excitation current and armature current

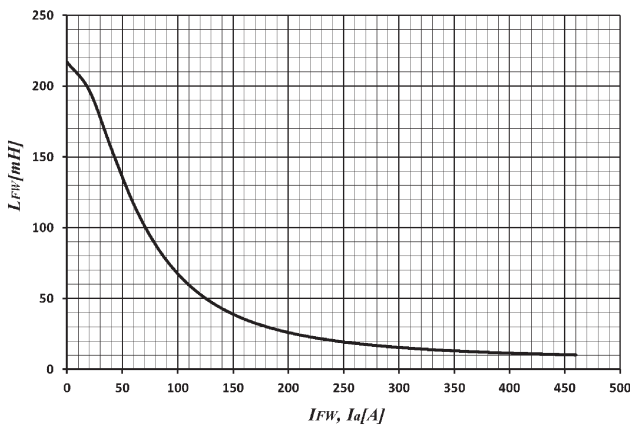


Fig. 6. The dependence of the inductance of field winding from the excitation current and armature current

We use the methodology described in [9, 13] for checking the adequacy of the calculations of inductive parameters. So as to determine the inductance of the armature winding is necessary to know the design parameters of the traction motor, this article can only use the method of determining the inductance of the field winding. The essence of this methodology is the

graphic differentiation of the characteristics of magnetization of the motor. In [3] it is stated that with this method we obtain in fact not  $L_{FW}$ , and  $L_{FW} + M_{FW-a}$ , therefore, to check the correctness of calculations is possible only if  $I_{FW} = I_a = 0$  A. After verification of the calculation was derived value of the inductance  $L_{FW}(0,0) = 216,85$  mH, which coincides with the results obtained by expression (20) and is shown in Table 4. As can be seen from table 4, the absolute value of the mutual inductance between the armature winding and the field winding is small compared to inductance of the field winding; however, it is commensurable with its own inductance of the armature winding. To assess the degree of magnetic coupling of these windings we use the famous expression for the coupling coefficient [4]:

$$K = \frac{M_{FW-a}}{\sqrt{L_{aq} L_{FW}}} \quad (25)$$

The changing of the coupling coefficient depending on the exciting current and armature current are shown in Figure 7.

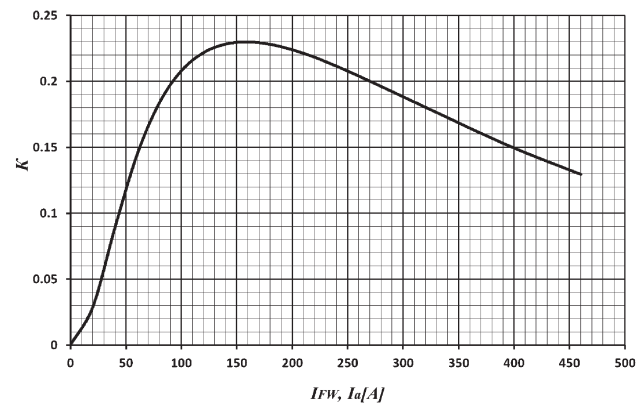


Fig. 7. The dependence of the coupling coefficient from the excitation current and armature current

As can be seen from Fig. 7, the value of the coupling coefficient at small currents is relatively small, which is the physical processes when a reaction anchors [8, 10, 16]. At these currents appears weak saturation of coil flux guide and armature reaction provides basically only a distorting effect on the magnetic flux. At currents close to the hour current, the coupling coefficient reaches a maximum, due to the fact that the edge of the pole under which there is an increase in induction is saturated, and the opposite edge is still on the unsaturated part of the curve of magnetization. With further increase of currents both edges of the pole are saturated, which causes a decrease in the difference of inductions and, as a consequence of the reduction of coupling coefficient.

## 7. Conclusions

The obtained expression for describing the magnetic flux of the electric machine  $\Phi_{dq}(F_{FW}, F_{aq}^*)$  allows the simulation when either the armature current for different degrees of attenuation of the excitation. On the basis of the expressions for  $\Phi_{dq}(F_{FW}, F_{aq}^*)$ , expressions for the inductive parameters of the electrical machine, which essentially depend on current are obtained. As shown by calculation we cannot neglect the inductive coupling between the excitation winding and the armature windings, especially at currents close to the hour current. Experimental analysis showed good agreement, which demonstrates the adequacy of the obtained expressions.

## Literature

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## Określenie parametrów indukcyjnych niezakończonych urządzeń DC biernych z uwzględnieniem reakcji armatury

### Streszczenie

Do obliczeń stanów nieustalonych w modelowaniu maszyn elektrycznych, należy określić indukcyjność uzwojeń. W tym celu należy znać parametry konstrukcyjne silnika, podstawową krzywą magnesowania lub bezpośrednio wykorzystać wyniki badań. Z reguły przy rozwiązywaniu problemu nie ma obliczonych danych dotyczących konkretnej maszyny. Ponadto, do analizy różnych warunków pracy, należy mieć równanie pozwalające na uzyskanie wartości parametrów indukcyjnych dla każdej możliwej wartości prądów płynących w cewkach silnika. Takie wyrażenie można uzyskać wykorzystując krzywą magnesowania maszyny pracującej bez obciążenia. Wiadomo, że podczas pracy maszyny nieskompensowanej, znaczący wpływ na wielkość strumienia magnetycznego ma reakcja twornika, która z kolei wpływa na jej indukcyjność. W tym przypadku, w celu określenia parametrów indukcyjności należy brać pod uwagę reakcję twornika. W artykule opisano metodę określania parametrów indukcyjnych nieskompensowanego silnika trakcyjnego z uwzględnieniem rozmagnesowującej reakcji twornika. Ta metoda umożliwi wykrywanie zależności analitycznych parametrów indukcyjnych, które mogą być bezpośrednio wykorzystane do symulacji elektromagnetycznych stanów nieustalonych w przypadku, gdy linearyzacja tych parametrów powoduje powstawanie dużych, niedopuszczalnych błędów w obliczeniach. W artykule nie uwzględniono wpływu prądów wirowych.

**Słowa kluczowe:** maszyna elektryczna prądu stałego, silnik trakcyjny, reakcja twornika, krzywa magnesowania, strumień magnetyczny biegunów, modelowanie

## Определение индукционных параметров машин постоянного тока с точки зрения реакции якоря

### Резюме

При моделировании электрических машин, для расчета переходных режимов, возникает необходимость в определении индуктивностей их обмоток. Данная задача может быть решена различными способами: зная конструктивные параметры машины, имея в распоряжении ее основную кривую намагничивания или непосредственно из опыта. Как правило, при решении задачи моделирования нет доступа к расчетным данным конкретного двигателя. Также, для исследования различных режимов работы, необходимо иметь выражение, позволяющее получить значение индуктивных параметров для любого возможного значения токов, протекающих по обмоткам двигателя. Такое выражение может быть получено, используя кривую намагничивания машины при холостом ходе. Известно, что при работе некомпенсированной машины существенное влияние на величину магнитного потока оказывает реакция якоря, что, в свою очередь, оказывает влияние и на ее индуктивности. В таком случае, при определении индуктивных параметров необходимо учитывать и действие реакции якоря. В работе описана методика определения индуктивных параметров некомпенсированного тягового электродвигателя с учетом размагничивающего действия реакции якоря. Данная методика позволяет получить аналитические выражения для индуктивных параметров, которые можно непосредственно использовать для моделирования переходных электромагнитных процессов в том случае, если линейаризация этих параметров вносит недопустимо грубую погрешность в расчеты. В работе не рассматривался учет влияния вихревых токов.

**Ключевые слова:** электрическая машина постоянного тока, тяговый электродвигатель, реакция якоря, кривая намагничивания, магнитный поток главных полюсов, моделирование